ADDITION AND SUBTRACTION (INCLUDING ALGEBRA)

# CURRICULUM PROGRESSION: TRACKING BACK

SINGLE-AGE CLASSES

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Titles in the series

Number and Place Value

**Addition and Subtraction (including algebra)**

Multiplication and Division (including algebra)

Fractions, Decimals and Percentages (including ratio and proportion)

Measurement

Geometry

Statistics

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# Guidance page

This document aims to show the progression in learning within key areas of mathematics and where the linked teaching can be found within ESSENTIALMATHS. Its purpose is to support teachers to track back to appropriate starting points for pupils who are not currently able to access age-appropriate learning so that they can make appropriate adaptations for them. This document should supplement the information provided on the front cover of each ESSENTIALMATHS sequence, which identifies how learning builds within and between sequences.

In many circumstances, teachers will use the tracking back information to support them in providing appropriate scaffolding for pupils up and into new learning. In addition, it is also particularly helpful when making adaptations for pupils who are operating further away from age related expectations including pupils with SEND so that a clear progression can be seen for their learning.

This document will allow teachers to track back from any ESSENTIALMATHS sequence to see how the learning builds from Early Years across Primary so that an appropriate starting point and progression can be identified for all pupils.

The teaching of addition and subtraction has been separated into nine, closely related strands of learning. These strands are:

- Part whole
- Equality
- Mental addition – regrouping
- Mental addition – reordering and rebalancing
- Mental subtraction – regrouping
- Mental subtraction – comparison and difference
- Mental calculation - using known facts and strategies
- Written addition
- Written subtraction

Within each strand, a short introduction outlines the focus of learning and highlights key building blocks and potential areas of weakness to support teachers in assessment and planning. A progression is then identified showing a breakdown of the small steps of learning and signposting where related teaching can be found within ESSENTIALMATHS referencing the learning sequence and step number. This includes links to earlier learning with Reception ESSENTIALMATHS and Essential Foundations for Counting (EFFC). Pre-requisite learning and opportunities for application are identified through highlighted rows in each progression.

**Pre-requisite learning - learning that will be built on within the progression**
The link may be across domains and therefore refer to another booklet in the series.
The link may be across strands and therefore refer to another strand in this booklet.

**Application**
Learning is often highly connected, and applications may be made across domains and contexts. Where this is the case, the sequence and step numbers are highlighted blue so that teachers can decide whether to explicitly make these links or continue through the progression at any one point.

# Part whole

Pupils need to be secure that numbers are built from other numbers. They should understand that a whole number can be broken (regrouped) into many parts and be clear that the sum of the parts is the total (whole) number. This means that if all the parts are known the whole number can be identified and an unknown part can be identified if the whole and other parts are known.

Part whole understanding supports pupils to be able to use inverse relationships to find unknown values. For example, 4 + □ = 10 can be worked out through understanding that the sum of two parts is the whole Therefore, if you take one of the parts from the whole, you can calculate the other part 10 – 4 = □. This ability to find unknown values underpins additive and comparative problem solving and will be used across a range of contexts to find missing values such as in statistics, measures and algebra.

Part whole understanding also links closely with the next strand **Equality** to support development of an understanding of commutativity: 4 + 3 = 3 + 4.

| **Pre-requisite learning** | **EFFC: Classification, Comparison, Pattern & Group recognition** | |
|---|---|---|
| **RLS10**<br>Regrouping the Whole | Step 1 | Identify parts within a given whole using subitising |
| | Step 2 | Regroup a whole into two parts using subitising, recognising that when combined, they equal the whole |
| | Step 3 | Identify that a whole can be broken into many parts |
| | Step 4 | Use subitising to identify multiple parts within a given whole |
| | Step 5 | Using a mixed set of objects to create a given whole, identify parts and provide reasoning for the groupings |
| **RLS11**<br>Regrouping parts to find the total (the whole) | Step 1 | Regrouping subitisable parts to make subitisable totals |
| | Step 2 | Regrouping subitisable parts and counting all to find the total |
| | Step 3 | Recognise that regrouping different combinations of parts can make the same whole |
| | Step 4 | Subitising one part and then counting on the number in the other part to find the whole |
| | Step 5 | Counting on from either part makes the same whole |
| **RLS12**<br>Finding the whole and missing parts | Step 1 | Finding the missing whole |
| | Step 2 | Missing part – what could it and what couldn't it be? |
| | Step 3 | Subitising the missing part |
| | Step 4 | Missing part – how many more? |
| | Step 5 | Missing part – how many left? |
| | Step 6 | Missing part – finding the difference |
| **1LS2**<br>Numbers to ten – finding patterns in numbers (including subitising) | Step 1 | Conservation of number |
| | Step 2 | Conservation of number - rearranging |
| | Step 3 | Subitising familiar patterns |
| | Step 4 | Subitising numbers to ten |

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| **1LS5**<br>Numbers to Ten – Regrouping the Whole | Step 1 | Represent numbers up to 10 in many ways through regrouping |
| | Step 2 | Matching values to mathematical models using increasingly complex regrouping |
| **1LS6**<br>Numbers to Ten – Part Whole Addition and Subtraction | Step 1 | Regrouping within numbers to 10 |
| | Step 2 | Exploring the language of addition |
| | Step 3 | Exploring commutativity |
| | Step 5 | Exploring ways to make 5 |
| | Step 6 | Using regrouping to make 5 and some more (think 5) |
| | Step 7 | Subtraction by taking away |
| | Step 8 | Explore the language of subtraction |
| **1LS7**<br>Numbers to Ten – Solving Problems Using Part or Whole Unknown | Step 1 | Revising identifying the whole and the parts (where all parts and wholes are shown) |
| | Step 2 | Story problems with unknown whole (addition) |
| | Step 3 | Story problems with one unknown part (subtraction – take away model) |
| | Step 4 | Matching representations |
| | Step 5 | Exploring statements focusing on language and proof |
| | Step 6 | Finding all possibilities |
| **1LS12**<br>Numbers to Twenty - Doubling and Halving | Step 1 | Building on part whole understanding where the parts are equal |
| | Step 2 | Replace colours with numbers and quantities to explore equal parts of the whole further |
| | Step 3 | Making doubles and finding halves using tens frames |
| **1LS20**<br>Numbers to twenty – part or whole unknown | Step 1 | Identifying the part and whole |
| | Step 2 | Identify if a part or the whole is missing |
| | Step 3 | Part whole relationships using +, - and = symbols |
| | Step 4 | Numbers to twenty - part or whole unknown |
| **1LS21**<br>Numbers to twenty – language of problem solving (part or whole unknown) | Step 1 | The language of problem solving (the whole as the result) |
| | Step 2 | The language of problem solving (a part as the result) |
| | Step 3 | Using the language of problem solving to solve problems with the whole unknown |
| | Step 4 | Using the language of problem solving to solve problems with a part unknown |
| | Step 5 | Developing the skills of problem solving |
| | Step 6 | Finding all possibilities |
| **1LS35**<br>Numbers to twenty - review | Step 6 | Worded problems for + and - |

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| **2LS10**<br>Finding part or whole unknown | Step 1 | Identifying the parts and the whole using Cuisenaire rods in a bar model |
| | Step 2 | Identifying the parts and whole in a cherry model |
| | Step 3 | Inverse relationship of addition and subtraction |
| | Step 4 | Using inverse to find missing numbers |
| | Step 5 | Using inverse to find missing numbers in problems |
| | Step 6 | Missing numbers in a range of contexts including measures |
| **2LS16**<br>Commutativity in addition but not subtraction | Step 1 | Reviewing the parts and the whole using Cuisenaire rods in a bar model |
| **2LS18**<br>Problem solving with addition and subtraction in a range of contexts | Step 1 | The language of problem solving |
| | Step 2 | Finding the unknown in a worded problem |
| | Step 3 | Choosing a strategy |
| | Step 4 | Strategies for solving missing number problems |
| | Step 5 | Further problem solving within statistics |
| **2LS21**<br>Double 1 and 2-digit Numbers | Step 1 | Doubling 2-digit numbers |
| **2LS34**<br>Problem solving for all operations, including fractions | Step 3 | Identifying the unknown |
| | Step 4 | Drawing to solve problems |
| **2LS38**<br>Mental calculation review | Step 2 | Identifying the unknown |
| | Step 3 | Checking using the inverse |
| **3LS7**<br>Fact families and applying the inverse | Step 2 | Creating fact families |
| | Step 3 | Using fact families and the inverse operation to find missing number |
| Addition and subtraction calculation strategies as appropriate:<br>**Mental addition – regrouping**<br>**Mental addition – reordering and rebalancing**<br>**Mental subtraction – regrouping**<br>**Mental subtraction – comparison and difference** | | |
| **3LS8**<br>Written addition | Step 6 | Language of addition |
| **3LS10**<br>Problem solving – worded problems | Step 1 | Identifying the part or whole unknown in simple worded problems |
| | Step 2 | Understanding start, change and result problems |
| | Step 3 | Mixed Practice |
| | Step 4 | Understanding multi-step part whole worded problems |
| | Step 5 | Understanding simple comparison problems |
| **3LS33**<br>Time - duration | Step 1 | Time to the nearest hour |
| | Step 5 | Finding unknown start and end times from given duration of events |
| | Step 6 | Comparing the duration of events |
| **Addition and Subtraction, Multiplication and Division –** Calculation strategies for all four operations | | |
| **4LS8** | Step 1 | Addition and subtraction problems involving measures |

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| Problem solving including measures to apply place value, mental strategies & arithmetic laws | Step 6 | Two step problems involving all four operations |
| **4LS11**<br>Measure – compare, estimate and calculate | Step 3 | Calculating with length, mass and capacity |
| **Fractions, Decimals and Percentages: Decimals** – Place value with decimal numbers | | |
| **4LS17**<br>Calculating with decimals | Step 1 | Finding complements to 1 |
| **4LS19**<br>Problem solving involving decimals to two decimal places | Step 1 | Non-routine problem-solving using decimals – using a simpler case to solve a complex problem |
| | Step 3 | Routine problem solving |
| **4LS26**<br>Time - read, write, calculate, convert | Step 3 | Find unknown start or end times when duration is known |
| **5LS16**<br>Problem solving - Four operations | Step 1 | Drawing a model to support reasoning |
| **5LS28**<br>Identify Unknown Angles | Step 1 | Angles in a right angle and on a straight line |
| | Step 2 | Angles around a point or whole turn |
| **5LS35**<br>Solve problems involving the four operations | Step 1 | Exploring confusing language – dangers of trigger words and distractors |
| | Step 2 | Focus on structure – translating language into a mathematical model |
| **5LS37**<br>Use Properties of Rectangles | Step 1 | Calculating missing lengths in rectangles and shapes or patterns including rectangles |
| | Step 2 | Using knowledge of rectangles and angles to calculate missing angles rectangles |
| **6LS4**<br>Problem solving (four operations) | Step 1 | Using a bar model to solve multi-step problems |
| **6LS16**<br>Order of Operations | Step 1 | Why we need the order of operations |
| | Step 2 | Develop order of operations and start to write formulas |
| | Step 3 | Deepen understanding of order of operations – abstract calculations |
| | **Multiplication and Division: Multiplication facts** – Square and cube | |
| | Step 4 | Considering division and indices (powers) in order of operations |
| | Step 6 | Connecting algebraic equations to known models (multiplication and division) |
| | Step 7 | Simplifying equations to find the unknown |
| | Step 8 | Solving word problems involving algebra |
| | Step 9 | Solving problems involving algebra – abstract calculations |
| **6LS18**<br>Exploring Relationships Between Perimeter and Area | Step 1 | Consolidate understanding of perimeter |
| | Step 2 | Consolidate area -rectilinear shape, triangle, parallelogram |
| **6LS19**<br>Recognise and Find Angles | Step 1 | Recognise and name angles (expressing algebraically) |
| | Step 2 | Investigate vertically opposite angles |
| | Step 3 | Find missing angles from known facts |

# Equality

Understanding equality is crucial for pupils to be able to recognise when quantities or expressions have the same value. This links closely to the previous strand **Part whole** knowledge as it enables pupils to understand commutativity: 4 + 3 = 3 + 4 and that numbers can be regrouped in different ways but have the same value: 5 + 1 = 2 + 4. This links with **Number and place value: Place value** understanding. For example, pupils are secure that 17 ones is equal to 1 ten and 7 ones and can use this to flexibly regroup numbers and to develop efficient calculation strategies.

When considering equality, it is important that pupils are also able to recognise and express inequality accurately.

| **Pre-requisite learning** | **EFFC: Comparison & Group recognition** | |
|---|---|---|
| **RLS1**<br>Subitising including equivalence, more and less | Step 1 | Recognising familiar patterns to subitise, noticing when the patterns are the same and different |
| | Step 2 | Making the same values in different patterns to subitise |
| | Step 3 | Noticing when something has a different value when subitising |
| **RLS7**<br>Using counting to compare | Step 1 | Use one to one correspondence to compare quantities |
| **RLS10**<br>Regrouping the Whole | Step 2 | Regroup a whole into two parts using subitising, recognising that when combined, they equal the whole |
| | Step 3 | Identify that a whole can be broken into many parts |
| | Step 4 | Use subitising to identify multiple parts within a given whole |
| **RLS11**<br>Regrouping parts to find the total (the whole) | Step 5 | Counting on from either part makes the same whole |
| **1LS2**<br>Numbers to ten – finding patterns in numbers (including subitising) | Step 1 | Conservation of number |
| | Step 2 | Conservation of number - rearranging |
| **1LS6**<br>Numbers to Ten – Part Whole Addition and Subtraction | Step 3 | Exploring commutativity |
| | Step 5 | Exploring ways to make 5 |
| | Step 9 | Subtraction is not commutative |
| **1LS9**<br>Numbers to Ten – Equality and Balance | Step 1 | Using language to express equivalent ways of making the same total |
| | Step 2 | Using language to express equivalent ways of making the same total (using a tens frame) |
| | Step 3 | Making equal values using symbols to record |
| | Step 4 | Making equivalent values using addition and subtraction |
| | Step 5 | Bonds to 10 |
| | Step 6 | Finding equivalents |

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| **1LS19**<br>Numbers to twenty – equality and balance | Step 1 | Explore different ways to total the same value (numbers 11 to 20) |
| | Step 2 | Exploring different ways to make the same total, including + and - (numbers 11 to 20) |
| | Step 3 | Bonds to 20 |
| **1LS23**<br>Measures – Coins and Combinations to 20p, Ordering and Comparing | Step 3 | Calculating coin combinations for values that do not have a designated coin below 10p |
| | Step 4 | Calculating coin combinations for values that do not have a designated coin between 11p and 20p |
| | Step 5 | Compare and order different combinations of coins |
| **1LS35**<br>Numbers to twenty - review | Step 2 | Equality |
| | Step 3 | Inequality |
| **2LS1**<br>Numbers to 100: counting, place value, ordering and comparing | Step 5 | Equivalence (=) and inequality (< and >) |
| **2LS11**<br>Money – making combination and finding change | Step 1 | Find different combinations of coins that equal the same amounts of money |
| **2LS16**<br>Commutativity in addition but not subtraction | Step 2 | Prove that addition is commutative |
| | Step 3 | Prove that commutativity is not possible when subtracting |
| **3LS5**<br>Mental fluency - addition | Step 5 | Understanding sum and commutativity in addition |
| **3LS6**<br>Mental fluency – subtraction | Step 1 | Subtraction is not commutative |
| **3LS7**<br>Fact families and applying the inverse | Step 1 | Commutative or not commutative |
| | Step 4 | Solving more complex missing number problems |
| **6LS16**<br>Order of Operations | Step 7 | Simplifying equations to find the unknown |
| **6LS28**<br>Algebra and sequences | Step 3 | Find pairs of numbers that satisfy an equation with two unknown variables |

# Mental addition - regrouping

Regrouping is one strategy that can be used to support mental addition calculations. This builds upon **Part whole** understanding and knowing that numbers can be regrouped in many ways. Pupils will be taught to regroup either addend to support efficiency so that they can bridge through close benchmarks and build on facts they already know. Pupils will need to have secure **Number and place value: Magnitude** so that they know the near benchmarks.

Once understood, regrouping strategies can be applied to other contexts such as regrouping time (base 60) and regrouping fractions to be able to support efficiency and to bridge through important benchmarks.

| **Pre-requisite learning** | **EFFC: All areas** | |
|---|---|---|
| | **Reception: RLS10-** Regrouping the Whole, **RLS11 -** Regrouping parts to find the total (the whole) | |
| | **Part whole understanding**: Ability to regroup numbers within 10 | |
| **1LS6**<br>Numbers to Ten – Part Whole Addition and Subtraction | Step 6 | Using regrouping to make 5 and some more (think 5) |
| **1LS17**<br>Number to twenty – adding using 'think 10' | Step 1 | Make 10 and using think 5 recap |
| | Step 2 | Think 10 by regrouping the second addend |
| | Step 3 | Think 10 by regrouping the first addend |
| | Step 4 | Think 10 when regrouping a two-digit number to aid addition |
| | Step 5 | Using think 15 |
| **2LS7**<br>Numbers to 20: mental addition and subtraction | Step 1 | Using 10 for adding 3 single digit numbers |
| | Step 2 | Adding more than two single digit numbers using reordering |
| **2LS9**<br>Add and subtract numbers mentally using 1- and 2-digit numbers | Step 3 | Finding the nearest multiple of ten |
| | Step 8 | Regrouping to "think 10" in addition |
| | Step 9 | Adding a 1-digit number to a 2-digit number using think 10 |
| | Step 10 | Adding a 2-digit number to a 2-digit number using think 10 |
| **3LS2**<br>Counting on and back in ones, tens and hundreds | Step 1 | Count on and back in tens with 2-digit numbers (crossing 100) |
| | Step 2 | Count on and back in tens and hundreds (2- and 3- digit numbers) |
| | Step 3 | Regrouping through hundreds |
| | Step 4 | Counting on and back in ones, tens and hundreds including regrouping |

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| **3LS5**<br>Mental fluency - addition | Step 1 | Adding 3-digit numbers to ones, tens and hundreds with no regrouping |
| | Step 2 | Adding 3-digit numbers to ones with regrouping ('Think 10') |
| | Step 3 | Adding 2- and 3-digit numbers to tens with regrouping ('Think 100') |
| | Step 4 | Mental addition with 2- and 3-digit numbers |
| **3LS33**<br>Time - duration | Step 1 | Time to the nearest hour |
| | Step 2 | Adding hours and minutes |
| | Step 4 | Duration of time |
| | Step 5 | Finding unknown start and end times from given duration of events |
| **4LS3**<br>Securing addition and subtraction mental fluency | Step 3 | Extending regrouping 'Think 100' and 'Think 1000' to adding 3- and 4-digit numbers |
| **4LS11**<br>Measure – compare, estimate and calculate | Step 4 | Calculating time addition (hours and minutes) |
| | Step 6 | Calculating duration of time (hours and minutes) |
| **4LS17**<br>Calculating with decimals | Step 1 | Finding complements to 1 |
| | Step 2 | Regrouping for addition |
| **4LS18**<br>Measure - money | Step 1 | Calculating with money – mental and written addition |
| **4LS20**<br>Add and subtract fractions with the same denominator | Step 1 | Finding complements of 1 |
| | Step 2 | Add and subtract fractions with no regrouping |
| | Step 3 | Add fractions with regrouping |
| **4LS26**<br>Time – read, write, calculate and convert time | Step 2 | Understanding and calculating duration |
| | Step 3 | Find unknown start or end times when duration is known |
| **4LS29**<br>Negative Numbers – Counting through Zero and Calculating in Context | Step 3 | Solving simple problems involving negative numbers |
| **5LS2**<br>Interpret Negative Numbers | Step 3 | Application in context |
| **5LS9**<br>Add and subtract using a range of strategies | Step 3 | Regrouping to add and subtract |
| **6LS1**<br>Place Value | Step 5 | Negative numbers |

# Mental addition – reordering and rebalancing

Reordering can be used to make calculations more manageable. For example, by allowing pupils to reorder calculations to use known complements. Pupils will need to be secure in their understanding of **Equality** and **Part whole** to understand that changing the order of the parts in an addition calculation does not impact the sum (commutativity).

To be able to use compensation or rebalancing strategies, pupils need to have a good sense of **Number and place value: Magnitude**. This includes recognising close benchmarks and how these can be used to support efficient calculation. They will need secure understanding of sum to rebalance calculations, knowing that if one addend is increased by a value and the other is reduced by the same value an equal sum is maintained and the total value will not change. For example, 9 + 7 = 16 can be rebalanced to 10 ($^{+1}$) + 6 ($^{-1}$) = 16.

| **Pre-requisite learning** | **Equality:** Addition is commutative<br>**Part Whole:** Numbers can be regrouped in different ways with equal sum<br>**Number and Place Value – Magnitude:** Recognise position of numbers within the number system and close benchmarks | |
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| **2LS7**<br>Numbers to twenty – Mental addition and subtraction | Step 1 | Using 10 for adding 3, single digit numbers |
| | Step 2 | Adding more than two single digit numbers using reordering |
| | Step 3 | Rebalancing when adding 9 or 11 |
| **2LS9**<br>Add and subtract numbers mentally using 1- and 2-digit numbers | Step 3 | Finding the nearest multiple of ten |
| | Step 4 | Rebalancing for equal sum |
| | Step 5 | Using rebalancing in context |
| **3LS5**<br>Mental fluency - addition | Step 5 | Understanding sum and commutativity in addition |
| | Step 6 | Finding complements and reordering |
| | Step 7 | Using compensation to add |
| **4LS3**<br>Securing addition and subtraction mental fluency | Step 4 | Introducing equal sum as a mental strategy |
| **4LS17**<br>Calculating with decimals | Step 1 | Finding complements to 1 |
| **5LS9**<br>Add and subtract using a range of strategies | Step 4 | Using equal sum for addition |

# Mental subtraction - regrouping

Regrouping is one strategy that can be used to support mental subtraction calculations. This builds upon **Part whole** understanding and knowing that numbers can be regrouped in many ways. Pupils will be taught to regroup either the minuend or subtrahend to make the calculation easier, supporting efficiency and building on facts they already know.

Once understood, regrouping strategies can be applied to other contexts such as regrouping time (base 60) and regrouping fractions to be able to support efficiency and to bridge through important benchmarks.

| **Pre-requisite learning** | **EFFC: All areas** | |
|---|---|---|
| | **Reception: RLS10-** Regrouping the Whole, **RLS11 -** Regrouping parts to find the total (the whole) | |
| | **Part whole understanding**: Ability to regroup numbers within 10 | |
| **1LS6**<br>Numbers to Ten – Part Whole Addition and Subtraction | Step 1 | Regrouping within numbers to 10 |
| | Step 7 | Subtraction by taking away |
| **1LS18**<br>Numbers to twenty – subtraction using 'think 10' | Step 1 | Counting back from 20 |
| | Step 2 | Subtracting 1-digit numbers from 2-digit numbers, below twenty, without crossing 10 |
| | Step 3 | Subtracting 1-digit numbers from numbers between 10 - 20 crossing the benchmark 10 |
| | Step 4 | Subtracting 1-digit numbers from numbers between 10 - 20 by regrouping and taking from the 10 |
| **2LS7**<br>Numbers to twenty – Mental addition and subtraction | Step 5 | Using counting on and back through 10 to compare and calculate the difference |
| | Step 6 | Use think addition for subtraction |
| **2LS8**<br>Finding compliments of 10 and 100 including measures | Step 3 | Think addition for subtraction using multiples of 10 within the context of a problem |
| | Step 4 | Think addition for subtraction using multiples of 10 within measure |
| **2LS9**<br>Add and subtract numbers mentally using 1- and 2- digit numbers | Step 11 | Using "think 10" for subtraction |
| | Step 12 | Subtracting a 1-digit number from a 2-digit number using think 10 |
| **2LS12**<br>Money – making combination and finding change | Step 2 | Solve calculations involving subtraction of money of the same unit |
| **2LS17**<br>Written subtraction | Step 1 | Subtracting a 1-digit number from a 2-digit number – counting back using think 10 and regrouping the subtrahend |
| | Step 2 | Subtracting a 1-digit number from a 2-digit number – regrouping the minuend |
| | Step 3 | Subtracting tens from a 2-digit number |

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| **3LS6**<br>Mental fluency - subtraction | Step 2 | Place value subtraction |
| | Step 3 | Subtracting hundreds, tens and ones with no regrouping |
| | Step 4 | Subtracting ones from 2-digit numbers with regrouping |
| | Step 5 | Subtracting multiples of ten from 3-digit number with regrouping |
| | Step 6 | Subtracting 2-digit numbers from 2-digit numbers with regrouping |
| **3LS33**<br>Time - duration | Step 1 | Time to the nearest hour |
| | Step 3 | Subtracting hours and minutes |
| | Step 4 | Duration of time |
| | Step 5 | Finding unknown start and end times from given duration of events |
| **4LS3**<br>Securing addition and subtraction mental fluency | Step 5 | Regrouping the minuend (the number being reduced) for subtraction |
| **4LS11**<br>Measure – compare, estimate and calculate | Step 5 | Calculating time subtraction (hours and minutes) |
| | Step 6 | Calculating duration of time (hours and minutes) |
| **4LS17**<br>Calculating with decimals | Step 3 | Regrouping for subtraction |
| **4LS18**<br>Measure - money | Step 2 | Calculating with money – mental and written subtraction |
| **4LS20**<br>Add and subtract fractions with the same denominator | Step 1 | Finding complements of 1 |
| | Step 2 | Add and subtract fractions with no regrouping |
| | Step 4 | Subtract fractions with regrouping |
| **4LS26**<br>Time – read, write, calculate and convert time | Step 2 | Understanding and calculating duration |
| | Step 3 | Find unknown start or end times when duration is known |
| **5LS9**<br>Add and subtract using a range of strategies | Step 2 | Using place value to add and subtract |
| | Step 3 | Regrouping to add and subtract |

# Mental subtraction – comparison and difference

Mental subtraction calculations can be supported by using comparison and difference. Pupils will need to be confident in the language of comparison and understand that difference is a measure of how far apart numbers are in the number system and, therefore, the result of subtracting one number from another. This means that **Number and place value: Magnitude** understanding needs to be secure.

As understanding of difference grows, it can be used to support mental subtraction, by rebalancing calculations and keeping an equal difference. Pupils need to be secure that equal difference is maintained if the same value is added or subtracted from the minuend and subtrahend: Minuend – Subtrahend = Difference. Pupils can then use this to make calculations more manageable mentally and to build on their known facts. For example, 28 - 13 can be rebalanced to 30 ($^{+2}$) - 15 ($^{+2}$) = 15 or 25 ($^{-3}$) - 10 ($^{-3}$) = 15.

| **Pre-requisite learning** | **EFFC: Classification and Comparison**<br>**Reception: RLS3** - Comparison - Measures, **RLS7** – Using counting to compare | |
|---|---|---|
| **1LS8**<br>Numbers to Ten – Comparison | Step 1 | Using 1:1 correspondence to find how many more / fewer |
| | Step 2 | Introducing the language of difference |
| | Step 3 | Understand difference as the distance between two numbers |
| | Step 4 | Finding difference in context |
| **1LS22**<br>Numbers to Twenty – Comparison (difference, more, less, fewer) including Statistics | Step 1 | Comparing values using 1:1 and familiar structures |
| | Step 2 | Finding the difference between values |
| | Step 3 | Finding the difference in the context of statistics |
| | Step 4 | Solving problems involving comparison and difference |
| **2LS7**<br>Numbers to twenty – Mental addition and subtraction | Step 4 | Rebalancing when subtracting 9 or 11 |
| | Step 5 | Using counting on and back through 10 to compare and to calculate difference |
| | Step 6 | Use think addition for subtraction |
| **2LS8**<br>Finding compliments of 10 and 100 including measures | Step 3 | Think addition for subtraction using multiples of 10 - problems |
| | Step 4 | Think addition for subtraction using multiples of 10 - measures |
| **2LS9**<br>Add and subtract numbers mentally using 1- and 2- digit numbers | Step 3 | Finding the nearest multiple of ten |
| | Step 6 | Difference |
| | Step 7 | Rebalancing to find equal difference |
| **2LS12**<br>Comparison (difference, more, less, fewer) | Step 1 | Understand difference when comparing on number lines |
| | Step 2 | Compare values in the context of measures (g) |
| | Step 3 | Compare values in the context of measures (Kg) |
| **3LS6**<br>Mental fluency - subtraction | Step 8 | Compensation |
| **4LS3**<br>Securing mental fluency | Step 6 | Introducing equal difference for mental subtraction |
| **5LS9**<br>Add and subtract using a range of strategies | Step 5 | Using equal difference for subtraction |

# Mental calculation – using known facts and strategies

To become fluent, pupils need to understand and be able to use calculation strategies and also develop an awareness of when these are most appropriately applied. Initially, this will require pupils to be taught how to consciously select strategies and this will lead to later independence and efficiency. Pupils should build on their secure knowledge of facts and strategies and, therefore, pre-requisites for this teaching will depend on wider aims. For example, pupils may be taught to choose between strategies and would have to be secure in each strategy they are choosing from but may not have to be secure in all strategies that peers may be selecting from. Making strategy choices, reasoning about them and wider application supports development of secure conceptual understanding and are therefore necessary for all pupils.

In addition to strategy selection, this progression builds learning about using estimation and rounding to support pupils to become independent in avoiding common errors and checking the reasonableness of solutions. This applies understanding of **Number and place value: Magnitude**.

| **Pre-requisite learning** | Understanding of relevant calculation strategies as appropriate:<br>**Mental addition – regrouping**<br>**Mental addition – reordering and rebalancing**<br>**Mental subtraction – regrouping**<br>**Mental subtraction – comparison and difference** | |
|---|---|---|
| **1LS13**<br>Numbers to Twenty – Odd and Even Numbers | Step 1 | Explore odd and even numbers through the use of tens frames |
| | Step 2 | Explore the alternating pattern of odd and even in consecutive numbers using number rods |
| | Step 3 | Explore the odd and even number values on a number line |
| **1LS35**<br>Numbers to twenty - review | Step 4 | Using known addition facts to choose efficient calculation strategies |
| | Step 5 | Strategies for calculating subtraction |
| **2LS8**<br>Finding compliments of 10 and 100 including measures | Step 1 | Rehearsing the complements to 10 and comparing them to the complements to 100 |
| | Step 2 | Continue to rehearse complements to 10 and 100 whilst regrouping flexibly |
| | Step 3 | Think addition for subtraction using multiples of 10 within the context of a problem |
| | Step 4 | Think addition for subtraction using multiples of 10 within measure |
| **2LS9**<br>Add and subtract numbers mentally using 1- and 2- digit numbers | Step 1 | Doubles and near doubles |
| | Step 2 | Using doubles and near doubles |
| | Step 13 | Choosing a strategy |
| **2LS11**<br>Money – making combination and finding change | Step 2 | Solve calculations involving subtraction of money of the same unit |
| | Step 3 | Solve simple problems in a practical context involving addition and subtraction of money of the same unit |
| | Step 4 | Continue to solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change |
| **2LS15**<br>Written addition method | Step 1: | Choosing the appropriate mental strategy when adding a two-digit number and ones |
| | Step 2: | Adding two-digit numbers and tens using concrete resources and pictorial representations |
| **2LS34**<br>Problem solving for all operations, including fractions | Step 1 | Choosing an efficient strategy – addition and subtraction |

| | | |
|---|---|---|
| **2LS38**<br>Mental calculation review | Step 1 | Reasoning about addition |
| **2LS41**<br>Place value and written calculation review | Step 1 | Problem solving with addition |
| **2LS41**<br>Place value and written calculation review | Step 2 | Checking for mistakes in written addition and subtraction |
| **3LS5**<br>Mental fluency - addition | Step 2 | Mental addition with 2- and 3- digit numbers |
| | Step 8 | Using multiple strategies to add mentally |
| **3LS38**<br>Measure – measuring and problem solving | Step 4 | Adding and subtracting involving measures |
| **4LS3**<br>Securing addition and subtraction mental fluency | Step 1 | Mental strategy comparison – considering appropriateness and efficiency |
| | Step 2 | Developing estimation to support calculation |
| | Step 7 | Mixed addition and subtraction practice |
| **4LS17**<br>Calculating with decimals | Step 6 | Comparing strategies |
| **4LS18**<br>Measure - money | Step 1 | Calculating with money – mental and written addition |
| **5LS9**<br>Add and subtract using a range of strategies | Step 1 | Using rounding for estimation |
| | Step 2 | Using place value to add and subtract |
| | Step 6 | Selecting an appropriate strategy |
| **6LS3**<br>Choosing effective mental calculation strategies | Step 1 | Reasoning the efficiency of mental strategy |
| | Step 2 | Using estimation to check mental calculations |
| | Step 3 | Applying and combining mental strategies to solve problems |
| **6LS31**<br>Application of known facts and calculation strategies | Step 1 | Identifying what we already know and how to use it (arithmetic focus) |
| | Step 2 | Avoiding common errors when calculating mentally |
| **6LS16**<br>Order of Operations and Algebra | Step 1 | Why we need the order of operations |
| | Step 2 | Develop order of operations and start to write formulas |
| | Step 3 | Deepen understanding of order of operations – abstract calculations |
| | Step 4 | Considering division and indices (powers) in order of operations |
| | Step 6 | Connecting algebraic equations to known models (multiplication and division) |
| | Step 7 | Simplifying equations to find the unknown |
| | Step 8 | Solving word problems involving algebra |
| | Step 9 | Solving problems involving algebra – abstract calculations |
| **6LS18**<br>Exploring Relationships Between Perimeter and Area | Step 1 | Consolidate understanding of perimeter (expressing algebraically) |
| | Step 2 | Consolidate finding the area of rectilinear shapes, parallelograms and triangles (expressing algebraically) |

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|---|---|---|
| **6LS19**<br>Recognise and Find Angles | Step 1 | Recognise and name angles (expressing algebraically) |
| | Step 2 | Investigate vertically opposite angles (expressing algebraically) |
| | Step 3 | Find missing angles from known facts (expressing algebraically) |
| **6LS20**<br>Reflection and Translation | Step 3 | Translate shapes in all four quadrants (expressing algebraically) |
| **6LS25**<br>Volume | Step 1 | Visualise and calculate the volume of cubes (expressing algebraically) |
| **6LS28**<br>Algebra and Sequences | Step 1 | Build and describe linear sequences |
| | Step 2 | Identify missing terms – start and end number given |
| | Step 3 | Find pairs of numbers that satisfy an equation with two unknown variables |
| **6LS34**<br>Further Algebra | Step 1 | Building sequences to generalise |
| | Step 2 | Linking sequences and algebra |
| | Step 3 | Describe the relationship between term and term number |

# Written addition

**Number and place value: Place value** understanding is crucial to written addition. Pupils need to be secure in the value of digits so that calculations can be correctly set up. They also need to be secure in understanding the Base 10 system and the regroup that occurs when 10 is reached within any place value column. Pupils will also need to be secure in mental calculation strategies for addition within 20. Once pupils are secure with written addition, this should become another strategy to select from when making choices.

| **Pre-requisite learning** | Mental calculation strategies for addition for numbers within 20<br>**Mental addition – regrouping**<br>**Mental addition – reordering and rebalancing** | |
|---|---|---|
| **2LS15**<br>Written addition method | Step 2 | Add two-digit numbers and tens using concrete resources and pictorial representations |
| | Step 3 | Add two 2-digit numbers using a written method with no regrouping |
| | Step 4 | Add two 2-digit numbers using a written method with regrouping |
| **2LS41**<br>Written calculation review | Step 2 | Checking for mistakes in written addition and subtraction |
| **3LS8**<br>Written addition | Step 1 | Columnar recording related to place value |
| | Step 2 | Formal written method with no regrouping (exchange) |
| | **Number and Place Value:** Regrouping 10 ones for 1 ten | |
| | Step 3 | Formal written method with regrouping of ones |
| | Step 4 | Regrouping tens and ones |
| | Step 5 | Using measurement units within addition |
| | Step 6 | Language of addition |
| **3LS38**<br>Measure – measuring and problem solving | Step 4 | Adding and subtracting involving measures |
| **4LS4**<br>Securing addition and subtraction written fluency | Step 1 | Formal addition method with no regrouping (thousands, hundreds, tens and ones) |
| | Step 2 | Formal addition method with regrouping in hundreds, tens & ones |
| | Step 3 | Formal addition method with regrouping in hundreds, tens and ones causing a further thousand |
| | Step 4 | Finding missing numbers in formal written addition |
| | Step 8 | Mixed practice |
| **4LS17**<br>Calculating with decimals | Step 4 | Formal written addition |
| | Step 6 | Comparing strategies |
| **4LS18**<br>Measure - money | Step 1 | Calculating with money – mental and written addition |
| **5LS10**<br>Add and subtract using formal written methods | Step 1 | Using rounding for estimation |
| | Step 2 | Column addition |
| | Step 4 | Reasoning about column addition and subtraction |

# Written subtraction

**Number and place value: Place value** understanding is crucial to written subtraction. Pupils need to be secure in the value of digits so that calculations can be correctly set up. They also need to be secure in understanding the Base 10 system and the regroup that occurs when 10 is crossed within any place value column. Pupils will also need to be secure in mental calculation strategies for subtraction within 20.

Once pupils are secure with written subtraction, this should become another strategy to select from when making choices.

| **Pre-requisite learning** | Mental calculation strategies for addition for numbers within 20<br>**Mental subtraction – regrouping**<br>**Mental subtraction – comparison and difference** | |
|---|---|---|
| **2LS17**<br>Written subtraction | Step 4 | Subtracting a 2-digit number from a 2-digit number with no regrouping |
| | Step 5 | Subtracting a 2-digit number from a 2-digit number with regrouping |
| **2LS41**<br>Place value and written calculation review | Step 2 | Checking for mistakes in written addition and subtraction |
| **3LS9**<br>Written subtraction | Step 1 | Formal written method with no regrouping (exchange) |
| | Step 2 | Subtraction – regrouping (exchange) tens into ones only |
| | Step 3 | Subtraction – regrouping hundreds into tens only |
| | Step 4 | Subtraction – regrouping hundreds and tens |
| | Step 5 | Missing number subtraction problems |
| | Step 6 | Mixed and multi-strategy practice |
| **3LS38**<br>Measure – measuring and problem solving | Step 4 | Adding and subtracting involving measures |
| **4LS4**<br>Securing addition and subtraction written fluency | Step 5 | Revisiting formal written subtraction (decomposition) |
| | Step 6 | Formal written subtraction with regrouping of thousands (decomposition) |
| | Step 7 | Missing number and written subtraction problems |
| | Step 8 | Mixed practice |
| **4LS17**<br>Calculating with decimals | Step 5 | Formal written subtraction |
| | Step 6 | Comparing strategies |
| **4LS18**<br>Measure - money | Step 2 | Calculating with money – mental and written subtraction |
| **5LS10**<br>Add and subtract using formal written methods | Step 1 | Using rounding for estimation |
| | Step 3 | Column subtraction |
| | Step 4 | Reasoning about column addition and subtraction |