MULTIPLICATION AND DIVISION (INCLUDING ALGEBRA)

CURRICULUM PROGRESSION: TRACKING BACK

SINGLE-AGE CLASSES

Titles in the series

Number and Place Value

Addition and Subtraction (including algebra)

Multiplication and Division (including algebra)

Fractions, Decimals and Percentages (including ratio and proportion)

Measurement

Geometry

Statistics

Guidance page

This document aims to show the progression in learning within key areas of mathematics and where the linked teaching can be found within ESSENTIALMATHS. Its purpose is to support teachers to track back to appropriate starting points for pupils who are not currently able to access age-appropriate learning so that they can make appropriate adaptations for them. This document should supplement the information provided on the front cover of each ESSENTIALMATHS sequence, which identifies how learning builds within and between sequences.

In many circumstances, teachers will use the tracking back information to support them in providing appropriate scaffolding for pupils up and into new learning. In addition, it is also particularly helpful when making adaptations for pupils who are operating further away from age related expectations including pupils with SEND so that a clear progression can be seen for their learning.

This document will allow teachers to track back from any ESSENTIALMATHS sequence to see how the learning builds from Early Years across Primary so that an appropriate starting point and progression can be identified for all pupils.

The teaching of multiplication and division has been separated into five closely related strands of learning. These strands are:

- Multiplication facts
- Understanding and calculating multiplication
- Understanding and calculating division
- Multiplying and dividing by 10, 100 and 1000
- Multiplication and division problem solving

Within each strand, a short introduction outlines the focus of learning and highlights key building blocks and potential areas of weakness to support teachers in assessment and planning. A progression is then identified showing a breakdown of the small steps of learning and signposting where related teaching can be found within ESSENTIALMATHS referencing the learning sequence and step number. This includes links to earlier learning with Reception ESSENTIALMATHS and Essential Foundations for Counting (EFFC). Pre-requisite learning and opportunities for application are identified through highlighted rows in each progression.

Pre-requisite learning - learning that will be built on within the progression.
The link may be across domains and therefore refer to another booklet in the series.
The link may be across strands and therefore refer to another strand in this booklet.

Application
Learning is often highly connected, and applications may be made across domains and contexts. Where this is the case, the sequence and step numbers are highlighted blue so that teachers can decide whether to explicitly make these links or continue through the progression at any one point.

Multiplication Facts

Building knowledge of multiplication facts is extremely closely linked to understanding of multiplication and ideally these should develop in parallel. Multiplication facts will develop from pupil understanding of pattern and repeated groups. Initially pupils will focus on doubling and halving groups, and this leads to skip counting. Pattern spotting should be encouraged, and the use of reasoning encouraged as a step towards developing recall. Pupils need to be clear how multiplication facts are also division facts.

As multiplication facts become secure, learning will be deepened through connection in a variety of contexts most notably identification of factors and multiples linked to **Fractions, decimals and percentages: fractions**.

Pre-requisite learning	**EFFC: Pattern and Group Recognition**	
	R: Pattern recognition	
RLS14 Doubling and halving	Step 1	Learning to identify equal and unequal groups
	Step 2	Identify doubles and halves (including the context of pattern)
	Step 3	Explore the relationship between doubles and halves
	Step 4	Establish part, whole understanding where the parts are equal
1LS12 Numbers to Twenty – Doubling and Halving	Step 1	Building on part whole understanding where the parts are equal
	Step 2	Replace colours with numbers and quantities to explore equal parts of the whole further
	Step 3	Making doubles and finding halves using tens frames
1LS24 Counting in 2s, 5s 10s.	Step 1	Counting in 2s and spotting patterns
	Step 2	Counting in 5s and spotting patterns
	Step 3	Counting in 10s and spotting patterns
	Step 4	Counting with coins – 2p, 5p and 10p
2LS21 Double and Halve One and Two-digit Numbers and Amounts of Money	Step 1	Doubling two-digit numbers
	Step 2	Halving multiples of ten
	Step 3	Halving two-digit numbers
	Step 4	Doubling and halving in the context of money
2LS22 Times Tables – 2s, 5s and 10s. Patterns and Strategy (counting in 3s)	Step 1	Patterns and strategies for the 2 times table
	Step 2	Patterns and strategies for the 5- and 10-times tables
	Step 3	Counting in 3s
3LS16 Multiplication – 3-, 4- and 8-Times Tables including Counting	Step 1	Understand that counting up in multiples is also repeated addition
	Step 2	Learning multiplication facts through building arrays
	Step 3	Learning multiplication facts through visualising arrays (developing recall)
	Step 4	Developing counting strategies for 3x and 4x tables

3LS17 Division – 1, 2, 3-, 5-, 4- and 8-Times Tables	Step 1	Division by sharing using manipulatives
	Step 2	Division by grouping using manipulatives
	Step 3	Linking multiplication and division using arrays
	Step 4	Learning division facts through visualising arrays (developing recall)
	Step 5	Rehearsing division facts
4LS5 Counting in Multiples of 6, 7, 9, 25 and 1000	Step 1	Understand that counting up in multiples is also repeated addition
	Step 2	Extend counting in multiples knowledge to 25s
4LS6 Multiplication and Division Facts (Times Tables)	Step 1	Creating and regrouping arrays for multiplication (distributive law)
	Step 2	Learning multiplication facts through building arrays (developing recall)
	Step 3	Rehearsing and recalling multiplication facts; making links and spotting patterns
	Step 4	Rehearsing division facts
	Step 5	Laws of divisibility to help with division facts
	Step 6	Strategies for calculating multiplication facts
4LS7 Factor Pairs, Integer Scaling and Correspondence Problems	Step 1	Understanding and finding factors
4LS34 Multiplication and Division Review	Step 1	Times table review
	Step 3	Related times tables facts
4LS35 Area	Step 2	Relate finding area of rectilinear shapes to arrays up to 12 x 12
5LS5 Properties of Number – Multiples, Factors and Common Factors	Step 1	Identifying multiples
	Step 2	Comparing multiples and factors
	Step 3	Identifying all factors of a number
	Step 4	Identifying common factors
5LS6 Prime and Composite Numbers	Step 1	Identifying what makes a number prime
	Step 2	Prime or composite?
	Step 3	Building composite numbers from prime factors
5LS7 Multiply and Divide Mentally	Step 1	Revisit strategies for recalling known facts
5LS8 Solve Problems Involving Knowledge of Key Facts	Step 1	Working backwards
	Step 2	Find a starting point
5LS21 Volume and Capacity	Step 1	Square numbers and area
	Step 2	Build cube numbers
6LS5 Application of Factors, Multiples and Primes	Step 1	Clarify terminology relating to properties of number
	Step 2	Recognise common multiples
	Step 3	Apply knowledge of common multiples
	Step 4	Apply knowledge of factors and multiples

Understanding and Calculating Multiplication

The concept of multiplication builds from pattern and group recognition. Initially, pupils will make links to counting including skip counting and understand multiplication as repeated **addition**. Later, pupils develop understanding of the arithmetic laws (commutative, associative and distributive) so that they understand how multiplication can be calculated efficiently using mental and formal methods.

Pre-requisite learning	**EFFC: Pattern and Group Recognition**	
	R: Pattern recognition and doubling and halving	
1LS27 Multiplication – Repeated Addition and Arrays (number of groups and size of group)	Step 1	Counting and repeated addition
	Step 2	The language of multiplication
	Step 3	Repeated addition and arrays (2s)
	Step 4	Repeated addition and arrays (5s and 10s)
1LS28 Multiplication – Problem Solving (identifying the number of groups and size of the group)	Step 1	Finding the maths in a picture
	Step 2	Multiplying the maths in a picture
	Step 3	Multiplication and measure
1LS29 Multiplication – Scaling and Counting in 2s to 24	Step 1	Exploring scaling
	Step 2	Twice as long
	Step 3	Twice as many - patterns
	Step 4	Twice as many - recipe
2LS23 Multiplication – Multiples and Repeated Addition	Step 1	Linking repeated addition and multiples
	Step 2	Multiples and multiplication
	Step 3	Exploring arrays
2LS24 Multiplication – Number of Groups, Group Size and Product	Step 1	The language of multiplication
	Step 2	The commutativity of multiplication
	Step 3	Strategies to calculate multiplication facts – regrouping to multiply
2LS35 Multiplication and Division – Equality and Balance	Step 1	Equality in multiplication
	Step 2	Keeping the balance
	Step 3	Comparing calculations
3LS18 Multiplication – Strategy, Associative and Distributive Laws	Step 1	Doubling and halving
	Step 2	Halving two-digit numbers
	Step 3	Associative law
	Step 4	Distributive law up to 10 x 10
	Step 5	Distributive law for 2-digit numbers
Multiplication Facts and Multiplying and Dividing by 10, 100 & 100		
3LS26 Multiplication – Formal Written Multiplication	Step 1	Multiplying two-digit numbers by ones using distributive law (no regrouping)
	Step 2	Multiplying two-digit numbers by ones using distributive law (with regrouping)

	Step 3	Introducing short multiplication with no regrouping
	Step 4	Short multiplication with regrouping of ones into tens only
	Step 5	Short multiplication with regrouping of ones and tens
4L24 Multiply Two and Three-digit Numbers by a One-digit Number Using a Formal Written Layout	Step 1	Multiplying multiples of ten by one-digit numbers
	Step 2	Multiplying multiples of one hundred by one-digit numbers
	Step 3	Multiplying two and three-digit numbers by one-digit numbers using distributive law (with regrouping)
	Step 4	Formal written multiplication with no regrouping
	Step 5	Formal written multiplication with regrouping in one column
	Step 6	Formal written multiplication with regrouping in one or more columns
4LS34 Multiplication and Division Review	Step 4	Short multiplication review
5LS7 Multiply and Divide Mentally	Step 2	Use known multiplication facts to derive others
	Step 3	Doubling and halving to use known facts
	Step 4	Divisibly rules
	Step 5	Regrouping to support division
	Step 6	Select an appropriate strategy for mental multiplication or division
5LS11 Formal Written Method for Multiplication	Step 1	Revision of formal written method for a 2 or 3-digit number by a 1-digit number
	Step 2	Short multiplication of a 3- or 4-digit number by 1 digit
	Step 3	Long multiplication of a 3- or 4-digit number by 2-digits
	Step 4	Comparing long multiplication and short multiplication
	Step 5	Rehearsal and application of the formal written methods of short and long multiplication
5LS30 Strategies for Multiplication and Division (Mental and Written)	Step 3	Multiplication and division – developing strategy discussion and operational sense
6LS6 Formal Written Method of Multiplication	Step 1	Revision of short multiplication for a 3- or 4-digit number by a 1-digit number
	Step 2	Revision of long multiplication for a 3- or 4-digit number by a 2-digit number
	Step 3	Revision of short multiplication for a 3- or 4-digit number by a 2-digit number
	Step 4	Generating new facts from known facts
	Step 5	Formal written method of multiplication involving numbers with up to 2 decimal places multiplied by a 1-digit number
	Step 6	Application of the formal written method for multiplication

Understanding and Calculating Division

The concept of division builds from pattern and recognition of equal groups. As understanding builds, pupil will consider how to express remainders based on context. Pupils need to understand division as both sharing and grouping. Links are made to multiplication. This is especially important when pupils begin to make use of multiplication facts to support increasingly efficient methods of division.

Pre-requisite learning	**EFFC: Pattern and Group Recognition**	
	R: Pattern recognition	
RLS14 Doubling and halving	Step 1	Learning to identify equal and unequal groups
	Step 4	Establish part, whole understanding where the parts are equal
RLS15 Odd and even	Step 1	Identify and recognise odd and even quantities by sharing into two groups
1LS26 Multiplication and Division – Equal or Unequal Groups and Remainders	Step 1	Sharing into equal groups
	Step 2	Sharing into unequal groups
	Step 3	Equal or unequal groups?
1LS30 Division – Sharing and Grouping Problems	Step 1	Sharing into equal groups
	Step 2	Solving sharing problems
	Step 3	Division by grouping
	Step 4	Solving grouping problems
2LS26 Division – Sharing and Grouping	Step 1	Division by sharing
	Step 2	Division by grouping
	Step 3	Division by grouping using arrays
	Step 4	Linking division and multiplication
	Step 5	Using multiplication facts to divide
2LS27 Division – Sharing and Grouping Problems including Remainders	Step 1	Patterns and rules of divisibility
	Step 2	Division with remainders – sharing
	Step 3	Division with remainders – grouping
	Step 4	Problems using division in context
	Step 5	Solving problems using division in context
2LS35 Multiplication and Division – Equality and Balance	Step 4	Using division to identify equality in multiplication

3LS17 Division – 1, 2, 3, 5, 4 and 8 Times Tables	Step 1	Division by sharing using manipulatives
	Step 2	Division by grouping using manipulatives
	Step 3	Linking multiplication and division using arrays
	Step 4	Learning division facts through visualising arrays (developing recall)
	Step 5	Rehearsing division facts
3LS28 Division – Two and Three-Digit Numbers by One-Digit Numbers including Halving	Step 1	Place value revision
	Step 2	Halving 2- and 3-digit numbers
	Step 3	Sharing 2- and 3-digit numbers by ones with no regrouping
	Step 4	Sharing 2- and 3-digit numbers by ones with regrouping
	Step 5	Linking base facts to division
3LS30 Division – Long Division	Step 1	Revision of quotients and remainders when sharing
	Step 2	Introducing the long division method (sharing ones)
	Step 3	Long division of tens and ones with no regrouping
	Step 4	Long division of tens and ones with regrouping
4LS25 Divide Two and Three-digit Numbers by a One-digit Number Using a Formal Written Layout	Step 1	Long division with no regrouping
	Step 2	Long division with regrouping hundreds into tens
	Step 3	Long division with regrouping hundreds into tens and tens into ones
	Step 4	Mixed division rehearsal
4LS34 Multiplication and Division Review	Step 5	Long division review
	Step 6	Short division
Multiplication Facts		
5LS12 Formal Written Method of Short Division	Step 1	Division as sharing
	Step 2	Sharing and grouping
	Step 3	Short division for numbers up to 4-digits
	Step 4	Expressing remainders as fractions
	Step 5	Expressing remainders as decimals
	Step 6	Interpreting remainders
5LS29 Formal Methods for Division and Multiplication-Increasingly Complex Problems	Step 1	Interpreting remainders
5LS30 Strategies for Multiplication and Division (Mental and Written)	Step 1	Revisiting and deepening understanding of remainders

6LS8 Formal Written Method of Short Division	Step 1	Understanding short division
	Step 2	Short division where answers have up to 2 decimal places
	Step 3	Short division with decimal remainders up to 2 decimal places
	Step 4	Prove decimal fraction equivalents using short division
6LS17 Formal Written Method for Long Division	Step 1	Comparing short and long division layout
	Step 2	Long division for numbers up to 4 digits
	Step 3	Interpreting remainders as whole numbers
	Step 4	Expressing remainders with fractions
	Step 5	Expressing remainders with decimals

Multiplying and Dividing by 10, 100 and 1000

Pupils need to develop a secure understanding of the effect of multiplying by powers of 10. This links to **Number and Place value: place value** understanding of the Base 10 number system. Pupil understanding is built through understanding of scaling initially with whole numbers and then moving into decimal numbers.

Metric **measurement: money, length, volume and capacity, mass** provides significant opportunities to develop further understanding of multiplication and division by powers of 10. Pupils may find it easier to start from the context of **measures** and then have explicit links made to the how this is replicated in our number system.

Pre-requisite learning	**EFFC: Pattern and Group Recognition**	
	R: Ten and some more	
	Place Value – understanding regrouping across place value columns	
3LS25 Multiplication – Multiplying Multiples of Ten	Step 1	Explore the effect of scaling by ten
	Step 2	Explore the effect of scaling by ten on place value
	Step 3	Multiplying multiples of ten by one-digit where the product is less than 100
	Step 4	Multiplying multiples of ten by one-digit where the product is greater than 100
3LS35 Place value and decimals - ten times bigger and ten times smaller	Step 1	Ten times smaller than 1 is a tenth
	Step 2	Recording tenths as decimal numbers
	Step 3	Finding unknown tenths from known wholes
	Step4	Finding unknown wholes from known tenths
4LS9 Multiply and Divide a One or Two-digit Number by 10 and 100	Step 1	Multiplying and dividing by 10 – investigating the effect
	Step 2	Multiplying and dividing by 10 – understanding the effect
	Step 3	Dividing by 10 – using decimal and fraction notation
	Step 4	Multiplying and dividing by 100 – understanding the effect, using decimal notation
	Step 5	Multiplying and dividing by 10 and 100 – applying learning and reasoning ideas
4LS10 Measure – Conversion of units	Step 1	Converting between units of length – understanding the calculations needed
	Step 2	Converting between units of mass and capacity – understanding the calculations needed
4LS24 Multiply Two and Three-digit Numbers by a One-digit Number Using a Formal Written Layout	Step 1	Multiplying multiples of ten by one-digit numbers
	Step 2	Multiplying multiples of one hundred by one-digit numbers
4LS34 Multiplication and Division Review	Step 3	Multiplying and dividing by 10/100 and 1000

5LS4 Multiply and Divide by 10, 100 and 1,000	Step 1	Multiplying by 10, 100 and 1000
	Step 2	Multiplying by 10, 100 and 1000 (including decimals)
	Step 3	Dividing by 10, 100 and 1000 (including decimals)
	Step 4	Multiplying and dividing by 10, 100 and 1000
5LS19 Measure – Converting Units of Measure	Step 1	Decimal and fraction equivalences of metric measures
	Step 2	Converting from a larger unit to a smaller unit
	Step 3	Converting from a smaller unit to a larger unit
	Step 4	Mixed conversion practice
5LS32 Imperial and Metric Conversions	Step 1	Metric conversion
	Step 2	Metric scale drawings
6LS2 Multiply and Divide by 10, 100 and 1,000	Step 1	Develop fluency of multiplying and dividing by 10, 100 and 1000
	Step 2	Application in the context of measure
6LS26 Measures	Step 1	Clarify what is known about measures and converting them

Multiplication and Division Problem Solving

Pupils will need to make links to a range of problems. This will include links to different contexts and consideration of different representations (concrete, pictorial and abstract including language variations and algebra). Specific context links such as **Area** and **Volume** can be found in the **Measurement** booklet.

Pre-requisite learning	**R: Characteristics of Effective Learning**	
2LS25 Multiplication Problem Solving	Step 1	Bar modelling for multiplication problems
	Step 2	Multiplication of measures
	Step 3	Multiplication and money (£ and p)
	Step 4	Mixed worded problems
3LS20 Multiplication and Division Worded Problems	Step 1	Worded problems based on equal groups
	Step 2	Rate worded problems involving money
	Step 3	Combination worded problems
	Step 4	Mixed bar model examples including measures and time
3LS27 Division Problem Solving – Sharing and Grouping	Step 1	Division by sharing – part whole problems
	Step 2	Division by sharing – comparison problems
	Step 3	Division by grouping
	Step 4	Using known facts to solve missing number problems
3LS29 Multiplication, Division and Fractions – Scaling and Correspondence Problems	Step 1	Solving integer scaling problems
	Step 2	Varying the unknown within correspondence problems
	Step 3	Mixed problems involving fractions
Addition and Subtraction, Multiplication and Division – Calculation strategies for all four operations		
3LS34 Securing the Four Operations with Whole Numbers including Problem Solving	Step 2	Applying multiplication and division including working systematically
4LS7 Factor Pairs, Integer Scaling and Correspondence Problems	Step 2	Solving integer scaling and correspondence problems
	Step 3	Exploring correspondence problems
	Step 4	Solving a range of correspondence problems
	Step 5	Creating their own correspondence problems
4LS37 Application and Problem Solving – Developing Operation Sense	Step 1	Number sequences
	Step 2	Number pattern and relationships
	Step 4	Solving logic problems
5LS16 Problem Solving – All Four Operations	Step 1	Draw a model to support reasoning
	Step 2	Interpreting statistical information
	Step 3	Working backwards as a strategy
	Step 4	Select an appropriate strategy to problem solve
	Step 5	Apply an appropriate strategy to problem solve

5LS19 Measure – Converting Units	Step 5	Scaling measures
5LS22 Percentages	Step 3	Use scaling to identify percentages
5LS29 Formal Methods for Division and Multiplication - Increasingly Complex Problems	Step 2	Creating word problems involving different division contexts
	Step 3	Applying formal multiplication to solve problems
5LS30 Strategies for Multiplication and Division	Step 2	Solving missing number division problems
5LS31 Solving Problems by Scaling by Simple Fractions and Rates	Step 1	Model scaling and correspondence problems
	Step 2	Scaling by simple fractions
	Step 3	Scaling by simple rates
	Step 4	Scale drawings
5LS32 Imperial and Metric Conversions	Step 3	Imperial units of measure - pints
	Step 4	Imperial units of measure - inches
	Step 5	Imperial units of measure - pounds
5LS35 Solving Problems involving the Four Operations	Step 1	Exploring confusing language – dangers of trigger words and distractors
	Step 2	Focus on structure – translating language into a mathematical model
	Step 3	What could the question be?
	Step 4	Revisiting working backwards
6LS16 Order of Operations and Algebra	Step 1	Why we need the order of operations
	Step 2	Develop order of operations and start to write formulas
	Step 3	Deepen understanding of order of operations – abstract calculations
	Multiplication and Division: Multiplication facts – square/cube number	
	Step 4	Considering division and indices (powers) in order of operations
	Step 6	Connecting algebraic equations to known models (multiplication and division)
	Step 7	Simplifying equations to find the unknown
	Step 8	Solving word problems involving algebra
	Step 9	Solving problems involving algebra – abstract calculations
6LS18 Exploring Relationships Between Perimeter and Area	Step 1	Consolidate understanding of perimeter (expressing algebraically)
	Step 2	Consolidate finding the area of rectilinear shapes, parallelograms and triangles (expressing algebraically)

6LS25 Volume	Step1	Visualise and calculate the volume of cubes (expressing algebraically)
6LS28 Algebra and Sequences	Step 1	Build and describe linear sequences
	Step 2	Identify missing terms – start and end number given
	Step 3	Find pairs of numbers that satisfy an equation with two unknown variables
6LS34 Further Algebra	Step 1	Building sequences to generalise
	Step 2	Linking sequences and algebra
	Step 3	Describe the relationship between term and term number